L2 GC _ S4

Exo 1: (E_1) EDL a coef cst $x = \frac{1}{3}$ est 1 sol particulière

$P(X) = 2X^2 - 5X + 3 = 2(X-1)(X-\frac{3}{2})$ Sol de (E_1) $y(x) = \frac{1}{3} + K_1 e^{x} + K_2 e^{\frac{3}{2}x}$

(E_2) Posons $\xi(u;v) = \xi(x+ay\,;y) = \varphi(x;y)$ $\quad \frac{\partial\varphi}{\partial x} = \frac{\partial\xi}{\partial u}$ $\quad \frac{\partial\varphi}{\partial y} = a\frac{\partial\xi}{\partial u} + \frac{\partial\xi}{\partial v}$.

φ est sol de $(E_2) \Leftrightarrow 2\frac{\partial\xi}{\partial u} + a\frac{\partial\xi}{\partial u} + \frac{\partial\xi}{\partial v} = 3\xi \underset{a=-2}{\Longleftrightarrow} \frac{\partial\xi}{\partial v} = 3\xi \Leftrightarrow \xi(u,v) = k(u)\,e^{3v}$

Les sol. de (E_2) sont les fonctions de la forme $\varphi(x;y) = K(x-2y)\,e^{3y}$

$\varphi(t,t) = K(-t)\,e^{3t} = \cos t$ donc $K(t) = \cos t\,.\,e^{3t}$ Finalement $\varphi(x;y) = \cos(x-2y)\,e^{3(x-y)}$

Exo 2: $\Phi(u,v) = P \Leftrightarrow \begin{cases} u+v=3 \\ u^2+v^2=5 \\ u^2-v^2=3 \end{cases} \Leftrightarrow \begin{cases} u+v=3 \\ u^2=4 \\ v^2=1 \end{cases} \Leftrightarrow \begin{cases} u+v=3 \\ u=\pm 2 \\ v=\pm 1 \end{cases} \Leftrightarrow \begin{cases} u=2 \\ v=1 \end{cases}$ $\quad P = \Phi(2;1)$.

2.2 $\Phi(u,v) = \Phi(2,1) + \frac{\partial\Phi}{\partial u}(2,1)(u-2) + \frac{\partial\Phi}{\partial v}(2,1)(v-1) + o(\|u-2;v-1\|) = P + (u-2)\begin{pmatrix}1\\4\\4\end{pmatrix} + (v-1)\begin{pmatrix}1\\2\\-2\end{pmatrix} + o(\|u-2;v-1\|)$

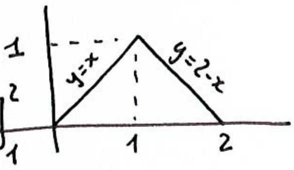
Le plan tangent à $\operatorname{Im}\Phi$ en P est $\operatorname{Im} L$ avec $L(u,v) = (u-2)\begin{pmatrix}1\\4\\4\end{pmatrix} + (v-1)\begin{pmatrix}1\\2\\-2\end{pmatrix} + P$.

Les vecteurs $\left(\begin{pmatrix}1\\4\\4\end{pmatrix};\begin{pmatrix}1\\2\\-2\end{pmatrix}\right)$ forment une base du plan tangent à Σ en P.

2.3 $\vec{n} = \begin{pmatrix}1\\4\\4\end{pmatrix} \wedge \begin{pmatrix}1\\2\\-2\end{pmatrix} = \begin{pmatrix}-16\\6\\-2\end{pmatrix}$ est un vecteur normal à Σ.

Equation du plan tangent à Σ en P : $-8(x-3) + 3(y-5) - (z-3) = 0$.

Exo 3: $I = \int_0^1\int_0^x \frac{1}{1+x}\,dy\,dx + \int_1^2\int_0^{2-x} \frac{1}{1+x}\,dy\,dx$



$= \int_0^1 \frac{x}{1+x}\,dx + \int_1^2 \frac{2-x}{1+x}\,dx = \int_0^1 1 - \frac{1}{1+x}\,dx + \int_1^2 \frac{3}{1+x} - 1\,dx = \left[x - \ln(1+x)\right]_0^1 + \left[3\ln(1+x) - x\right]_1^2$

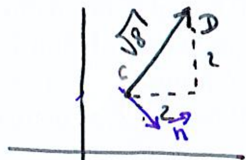
$= -\ln 2 + 3\ln 3 - 3\ln 2 = 3\ln 3 - 4\ln 2 = \ln\left(\frac{27}{16}\right)$

Exo 3.2: $A = \iint_D 1\,dx\,dy = \iint_D \frac{\partial Q}{\partial x}\,dx\,dy = \left|\int_{\partial D} Q(x,y)\,dy\right|$ avec $Q(x,y) = x$.

$dy = (1-2t)\,dt$

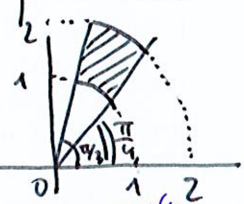
$= \left|\int_0^1 (t-t^3)(1-2t)\,dt\right| = \left|\int_0^1 t - 2t^2 - t^3 + 2t^4\,dt\right| = \left|\frac{1}{2} - \frac{2}{3} - \frac{1}{4} + \frac{2}{5}\right| = \left|\frac{30-40-15+24}{60}\right| = \frac{1}{60}$

Exo 4: $\overrightarrow{CD} = (2;2)$ $\qquad \vec{n} = \left(\frac{1}{\sqrt{2}};\frac{-1}{\sqrt{2}}\right)$ $\qquad \|\overrightarrow{CD}\| = \sqrt{8}$

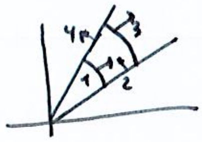


4.1: $F = (\Psi_1\vec{\imath} + \Psi_2\vec{\jmath})\,.\left(\frac{1}{\sqrt{2}}\vec{\imath} - \frac{1}{\sqrt{2}}\vec{\jmath}\right)\sqrt{8} = 2(\Psi_1 - \Psi_2)$

4.2: $\vec{\Phi} = \Phi(\tilde{x};\tilde{y}) + \frac{\partial\Phi}{\partial x}(\tilde{x};\tilde{y})(x-\tilde{x}) + \frac{\partial\Phi}{\partial y}(\tilde{x},\tilde{y})(y-\tilde{y}) + o(\|x-\tilde{x};y-\tilde{y}\|)$.

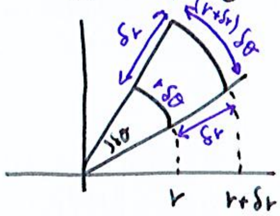


4.3:
4.4:



$F = \underbrace{-\Psi_r(r\,\delta\theta)}_{1} \underbrace{-\Psi_\theta\,\delta r}_{2} + \underbrace{(r+\delta r)\delta\theta\,\Psi_r}_{3} + \underbrace{\delta r\,\Psi_\theta}_{4}$

$= \Psi_r\,\delta r\,\delta\theta$.



4.5: Sur 1 $\;-\Psi_r(r,\tilde{\theta})\,r\,\delta\theta$

sur 2 $\;-\Psi_\theta(\tilde{r},\theta)\,\delta r$

sur 3 $\;\Psi_r(r+\delta r,\tilde{\theta})\,(r+\delta r)\,\delta\theta$

sur 4 $\;\Psi_\theta(\tilde{r},\theta+\delta\theta)\,\delta r$.

Longueurs des côtés de R_δ.

$F = \left(\Psi_r(r+\delta r,\tilde{\theta}) - \Psi_r(r,\tilde{\theta})\right) r\,\delta\theta + \Psi_r(r+\delta r,\tilde{\theta})\,\delta r\,\delta\theta + \left[\Psi_\theta(\tilde{r},\theta+\delta\theta) - \Psi_\theta(\tilde{r},\theta)\right]\delta r$

$= \frac{\partial\Psi_r}{\partial r}\,.\,\delta r\,r\,\delta\theta + \Psi_r\,\delta r\,\delta\theta + \frac{\partial\Psi_\theta}{\partial\theta}\,\delta\theta\,\delta r$

$= \left(\frac{\partial\Psi_r}{\partial r} + \frac{1}{r}\Psi_r + \frac{1}{r}\frac{\partial\Psi_\theta}{\partial\theta}\right)\underbrace{r\,\delta\theta\,\delta r}_{\text{aire de } R_\delta}$.