Mathématique 2 session 2.

Exo 1 : $\displaystyle\int_0^1 \frac{1}{(x-2)(x+2)}\,dx = \int_0^1 \frac{\frac{1}{4}}{x-2} + \frac{-\frac{1}{4}}{x+2}\,dx = \frac{1}{4}\Big[\ln|x-2| - \ln|x+2|\Big]_0^1 = \frac{1}{4}(-\ln 3) = \frac{-\ln 3}{4}.$

Exo 2 : $\theta = \arccos\left(\frac{1}{\sqrt{5}}\right)$ $\quad \cos\theta = \frac{1}{\sqrt{5}}$ $\quad \cos^2\theta + \sin^2\theta = 1$ donc $\sin^2\theta = \frac{4}{5}$ or $\theta \in]0;\pi[$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos x - 2\sin x = 0 \iff \frac{1}{\sqrt{5}}\cos x - \frac{2}{\sqrt{5}}\sin x = 0 \iff \cos\theta\cos x - \sin x\sin\theta = 0$

$\iff \cos(\theta + x) = 0 \iff (x+\theta) = \frac{\pi}{2} + k\pi \iff x = \frac{\pi}{2} - \theta + k\pi$

Exo 3 : $f(t) = \varphi(t^2; 1+t)$

$\varphi(1+h; 2+k) = \varphi(1,2) + \frac{\partial\varphi}{\partial x}(1,2)h + \frac{\partial\varphi}{\partial y}(1,2)k + \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial x^2}(1,2)h^2 + 2\frac{\partial^2\varphi}{\partial x\partial y}(1,2)hk + \frac{\partial^2\varphi}{\partial y^2}(1,2)k^2\right) + o(h^2+k^2)$

$= \varphi(1,2) + h - 2k + \frac{1}{2}(h^2 - 2hk + 0) + o(h^2+k^2)$

$f'(t) = 2t\frac{\partial\varphi}{\partial x}(t^2; 1+t) + \frac{\partial\varphi}{\partial y}(t^2; 1+t)$

$f''(t) = 2\frac{\partial\varphi}{\partial x}(t^2; 1+t) + 4t^2\frac{\partial^2\varphi}{\partial x^2}(t^2; 1+t) + 2t\frac{\partial^2\varphi}{\partial x\partial y}(t^2; 1+t) + 2t\frac{\partial^2\varphi}{\partial x\partial y}(t^2; 1+t) + \frac{\partial^2\varphi}{\partial y^2}(t^2; 1+t)$

$f'(1) = 0 \qquad f''(1) = 2 + 4 - 2 + (-2) + 0 = 2.$

$f(t) = \varphi(1;2) + \frac{1}{2}\cdot 2t^2 + o(t^2) = \varphi(1;2) + t^2 + o(t^2)$

φ ne possède pas d'extremum local en $(1,2)$ car $(1,2)$ n'est pas 1 point critique de φ.

f possède un minimum local strict en 1 car

$(f(t) - f(1)) = t^2\underbrace{(1 + o(1))}_{>0 \text{ pour } t \text{ petit}}$ $\quad$ donc $f(t) - f(1) > 0$ pour t petit.

Exo 4 : $4x + 4y + 4z = 4$ et $x > 0$ et $y > 0$ et $z > 0$

$V = xyz = xy(1-x-y) = xy - x^2y - xy^2$ avec $z = 1-x-y > 0$ donc $x+y < 1$.

$\frac{\partial\varphi}{\partial x} = y - 2xy - y^2 \quad \frac{\partial\varphi}{\partial y} = x - x^2 - 2xy.$

$\begin{cases} y(1-2x-y) = 0 \\ x(1-x-2y) = 0 \end{cases} \underset{x>0 \text{ et } y>0.}{\iff} \begin{cases} 1-2x-y=0 \\ 1-x-2y=0 \end{cases} \iff \begin{cases} 1-2x-y=0 \\ x-y=0 \end{cases} \iff \begin{cases} x = \frac{1}{3} \\ y = \frac{1}{3} \end{cases}$ un unique point critique sur T.

$\frac{\partial^2\varphi}{\partial x^2} = -2y \quad \frac{\partial^2\varphi}{\partial y^2} = -2x \quad \frac{\partial^2\varphi}{\partial x\partial y} = 1 - 2x - 2y$

$H = \begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$P_H(\lambda) = \left(-\frac{2}{3}-\lambda\right)\left(-\frac{2}{3}-\lambda\right) - \frac{1}{9} = \lambda^2 + \frac{4}{3}\lambda + \frac{1}{3} \qquad \Delta = \frac{16}{9} - \frac{4}{3} = \frac{4}{9} = \left(\frac{2}{3}\right)^2 \qquad r = \frac{-\frac{4}{3} \pm \frac{2}{3}}{2} < 0$

2 V.P négatives donc max local strict.