L2 GC / Mathématiques 2.

Exo1 Decomposition en éléments simples

$$\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2} \qquad \int_0^1 \frac{x}{(x+1)(x+2)}\,dx = \int_0^1 \frac{-1}{x+1} + \frac{2}{x+2}\,dx = \Big[-\ln(x+1)+2\ln(x+2)\Big]_0^1$$

$$= -\ln 2 + 2\ln 3 - 2\ln 2 = \ln\left(\frac{9}{8}\right).$$

Exo2 : 1. $\frac{\partial\varphi}{\partial x} = \frac{y(x^2+4y^2)-2x^2y}{(x^2+4y^2)^2} = \frac{y(4y^2-x^2)}{(x^2+4y^2)^2}$ $\qquad \frac{\partial\varphi}{\partial y}(x,y) = \frac{x(x^2+4y^2)-8xy^2}{(x^2+4y^2)^2} = \frac{x(x^2-4y^2)}{(x^2+4y^2)^2}$

2. $\frac{\varphi(x,0)-\varphi(0,0)}{x} = 0$ donc $\frac{\partial\varphi}{\partial x}(0,0)=0$ $\qquad \frac{\varphi(0,y)-\varphi(0,0)}{y-0} = 0 \xrightarrow[y\to 0]{} 0$ donc $\frac{\partial\varphi}{\partial y}(0,0)=0$

3. $\varphi(x,x) = \frac{1}{5} \xrightarrow{x\to 0} \frac{1}{5} \neq \varphi(0,0)$ donc φ n'est pas continue en $(0,0)$.

4. $\varphi(x,x) > 0$ pour tout $x \neq 0$ et $\varphi(x,-x) < 0$ pour $\forall x \neq 0$ donc pas d'extremum en $(0,0)$

Exo3 : $\frac{\partial\varphi}{\partial x} = 4x + e^{xz} + xz e^{xz} - 1$ $\qquad \frac{\partial\varphi}{\partial y} = 6y$ $\qquad \frac{\partial\varphi}{\partial z} = 2z + x^2 e^{xz}$

$\frac{\partial^2\varphi}{\partial x^2} = 4 + ze^{xz} + ze^{xz} + xz^2e^{xz}$ $\qquad \frac{\partial^2\varphi}{\partial x\partial y} = 0$ $\qquad \frac{\partial^2\varphi}{\partial y^2} = 6$ $\qquad \frac{\partial^2\varphi}{\partial y\partial z} = 0$ $\qquad \frac{\partial^2\varphi}{\partial z^2} = 2 + x^3e^{xz}$ $\qquad \frac{\partial^2\varphi}{\partial x\partial z} = (2x+zx^2)e^{xz}$

$$H_{(0,0,0)} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \varphi(x,y,z) = 2x^2 + 3y^2 + z^2 + (x^2+y^2+z^2)\,\varepsilon(x,y,z)$$

4. $$\varphi(x,y,z) - \varphi(0,0,0) = x^2+y^2+z^2 + (x^2+y^2+z^2)\,\varepsilon(x,y,z) = \underbrace{(x^2+y^2+z^2)}_{\substack{>0 \\ \text{pour } (x,y,z)\neq(0,0,0)}}\underbrace{(1+\varepsilon(x,y,z))}_{\substack{>0 \text{ pour } (x,y,z) \text{ assez} \\ \text{proche de } (0,0,0)}}$$

Exo4 : 1. $h : \mathbb{R} \to \mathbb{R}$

2. $\varphi(x;y) = \varphi(x_0,y_0) + \frac{\partial\varphi}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial\varphi}{\partial y}(x_0,y_0)(y-y_0) + \sqrt{(x-x_0)^2+(y-y_0)^2}\,\varepsilon(x,y).$

3. $f(t) = f(2) + f'(2)(t-2) + (t-2)\varepsilon(t)$ et $g(t) = g(2) + g'(2)(t-2) + (t-2)\varepsilon(t)$

$\quad = f'(2)(t-2) + (t-2)\varepsilon(t)$ $(\lim_2 \varepsilon = 0)$ $\quad = g'(2)(t-2) + (t-2)\varepsilon(t)$. $\quad \lim_2 \varepsilon = 0$

$\quad = (f'(2)+\varepsilon(t))(t-2)$ $\quad = (g'(2)+\varepsilon(t))(t-2)$

4. $$\frac{h(t)-h(2)}{t-2} = \frac{\varphi(f(t),g(t)) - \varphi(f(2),g(2))}{t-2} = \frac{\varphi\big((f'(2)+\varepsilon(t))(t-2);(g'(2)+\varepsilon(t))(t-2)\big) - \varphi(0,0)}{t-2}$$

5. $$\frac{h(t)-h(2)}{t-2} = \frac{\frac{\partial\varphi}{\partial x}(0,0)(f'(2)+\varepsilon(t))(t-2) + \frac{\partial\varphi}{\partial y}(0,0)(g'(2)+\varepsilon(t))(t-2) + \sqrt{(f'(2)+\varepsilon(t))^2+(g'(2)+\varepsilon(t))^2(t-2)^2}\,\varepsilon(t)}{t-2}$$

$$= \underbrace{\frac{\partial\varphi}{\partial x}(0,0)\,(f'(2)+\varepsilon(t)) + \frac{\partial\varphi}{\partial y}(0,0)(g'(2)+\varepsilon(t)) + \sqrt{(f'(2)+\varepsilon(t))^2+(g'(2)+\varepsilon(t))^2}\,\varepsilon(t)}_{\downarrow\, t\to 2}.$$

$$\frac{\partial\varphi}{\partial x}(0,0)\,f'(2) + \frac{\partial\varphi}{\partial y}(0,0)\,g'(2).$$

6. $h'(t) = \frac{\partial\varphi}{\partial x}(f(t);g(t))\,f'(t) + \frac{\partial\varphi}{\partial y}(f(t);g(t))\,g'(t).$

$$h''(t) = \frac{\partial^2\varphi}{\partial x^2}(f(t),g(t))\,(f'(t))^2 + \frac{\partial^2\varphi}{\partial y\partial x}(f(t),g(t))f'(t)g'(t) + \frac{\partial\varphi}{\partial x}(f(t),g(t))\,f''(t)$$
$$+ \frac{\partial^2\varphi}{\partial x\partial y}(f(t),g(t))\,f'(t)g'(t) + \frac{\partial^2\varphi}{\partial y^2}(f(t),g(t))(g'(t))^2 + \frac{\partial\varphi}{\partial y}(f(t),g(t))\,g''(t).$$