L1 - Bilingual Group - 2012/2013 - **15 May 2013** - Session 1

Examination in Mathematics for Sciences (MS2)

Duration: **3** hours – Documents, calculators, cell phones, smartphones and tablets are not allowed
The total score: 7+3+5+5 is given in an informal manner and may be modified

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Exercise 1 :

1. (a) Decompose the rational fraction $\dfrac{X^4-6}{(X+2)(X^2+1)}$ into simple elements on $\mathbb{R}$.

   (b) From this deduce the calculation of the integral : $I = \displaystyle\int_0^1 \frac{x^4-6}{(x+2)(x^2+1)}\,\mathrm{d}x$.

2. Give the domain of definition of the following primitives (antiderivatives), next find their expressions :

   (a) $\displaystyle\int \ln(x)\,\mathrm{d}x$, then $\displaystyle\int \ln(x)\ln(x^2)\,\mathrm{d}x$. (Hint : IBP)

   (b) $\displaystyle\int \frac{\sin x\ \mathrm{d}x}{\sin^2(x)\ -\ 5}$. (Hint : Bioche's rules)

Exercise 2 : One wants to find the set $\mathcal{S}$ of functions $y : \mathbb{R} \to \mathbb{R}$, solutions of the differential equation :

$$\text{(E)} \qquad y'' + 2y' - 3y = x\mathrm{e}^{2x}.$$

1. If (E_0) denotes the associated homogeneous equation, write down the characteristic equation and find the set $\mathcal{S}_0$ of solutions y_0 of (E_0).
2. Find the set $\mathcal{S}$ of solutions y of (E).

Exercise 3 : Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^2 + 2xy^2$. Let us denote by G_f the graph of f, namely, the set $G_f = \{(x,y,z) \in \mathbb{R}^3 \mid (x,y) \in \mathbb{R}^2 \text{ et } z = f(x,y)\}$.

1. Draw in the plane the curve of level $z = 0$ for f.
2. Compute the partial derivatives of f (to be denoted by $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$) in an arbitrary point $(x,y) \in \mathbb{R}^2$.
3. Give the expression of the gradient vector of f in such a point (to be denoted by $\mathrm{grad} f(x,y)$).
4. Find a cartesian equation of the tangent plane to G_f at the point $M(1;1;3)$.
5. Find the set of critical points of f.
6. By examining $f(-y^2, y)$, show that f does not have a local minimum in the origin.
7. Show that f does not have a local maximum in the origin.
8. Make a complete study for the extrema of f.

Exercise 4 : Consider the differential equation :

$$\text{(E)} \qquad x^3y' + 3x^2y = -\sin x$$

1) Find the set of solutions of (E) and give the intervals on which these solutions are valid.
2) Give the solution which satisfies $y(1) = 2$.
3) Does (E) have solutions on $\mathbb{R}$?

Questions related to the lecture.

1) Recall the definitions of the following functions and give the expression of their derivatives :

$$\textit{Arcsin}, \qquad \textit{Arccos}, \qquad \textit{Arctan}$$

2) Let $f : [0,1] \to \mathbb{R}$ be a function. Suppose f continuous, strictly increasing on $[0,1]$; let g be the inverse function of f (in particular, $\forall x \in [0;1],\ g(f(x)) = x$).

Let $\Phi : [0,1] \to \mathbb{R}$ be a function defined as follows :

$$\forall x \in [0,1], \quad \Phi(x) = \int_0^x f(t)\,\mathrm{d}t + \int_0^{f(x)} g(t)\,\mathrm{d}t - xf(x).$$

Show that if f is moreover differentiable then Φ is differentiable too and $\Phi' = 0$.
