Examination of Mathematics for Sciences (MS2)

Duration: **2** hours

Documents, calculators and mobile phones are forbidden

Exercice 1 : **a)** By using integration by parts (IBP) calculate one primitive of the application $f(x) = \arctan x$.

b) Let $g(x) = x \ln\left(\dfrac{1+x}{1-x}\right)$. Give the domain of definition of its primitives and next, calculate these primitives by using IBP.

Exercice 2 : We propose to calculate the integral $I = \displaystyle\int_0^{\ln 2} \frac{\mathrm{e}^x}{1+\mathrm{e}^{3x}}\,\mathrm{d}x$.

1.a) Let $P(t) = 1 + t^3$. Factorize this polynomial on $\mathbb{R}$.

1.b) Decompose the rational fraction $f(t) = \dfrac{1}{P(t)}$ into simple elements.

2) Deduce the value of the integral $J = \displaystyle\int_1^2 \frac{1}{1+t^3}\,\mathrm{d}t$.

3) Make a convenient change of variable in I that allows to calculate it in terms of J.

Exercice 3 : We aim to solve the *real* differential equation :

$$y'' - 2y' + 2y = \sin x + x\mathrm{e}^{2x}. \qquad \text{(E)}$$

1) Solve the homogeneous differential equation associated to (E), that is :

$$y'' - 2y' + 2y = 0. \qquad (\mathrm{E}_0)$$

Give the set of solutions as a real linear space ; give the dimension of this space.

2) Find a particular solution (that you denote y_1) for the following differential equation :

$$y'' - 2y' + 2y = \sin x. \qquad (\mathrm{E}_1)$$

3) Find a particular solution (that you denote y_2) for the following differential equation :

$$y'' - 2y' + 2y = x\mathrm{e}^{2x}. \qquad (\mathrm{E}_2)$$

4) Deduce the set of solutions of (E) (Give a brief justification).

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Exercice 4 : Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by $f(x,y) = xye^{x-y}$.

1) Calculate its partial derivatives (that we will denote by $\partial_x f \equiv \frac{\partial f}{\partial x}$ et $\partial_y f \equiv \frac{\partial f}{\partial y}$).

2) Show that f admits only two critical points, which are $(0,0)$ and $(-1,1)$.

3) Calculate the second order partial derivatives $\partial_{xx} f, \partial_{yx} f, \partial_{xy} f$ et $\partial_{yy} f$ at an arbitrary point (x,y).

4) Calculate the value of the determinant

$$\Delta(x,y) = \det \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}(x,y) & \frac{\partial^2 f}{\partial x \partial y}(x,y) \\ \frac{\partial^2 f}{\partial y \partial x}(x,y) & \frac{\partial^2 f}{\partial y^2}(x,y) \end{pmatrix}.$$

at each of the critical points $(0,0)$ and $(-1,1)$.

5) Deduce the nature of each of these critical points (local min, local max, saddle point)

Exercice 5 : [hors barème]

Show that $I = \displaystyle\int_A^B \frac{1}{x \cdot (\ln x) \cdot \ln(\ln x)}\, \mathrm{d}x = \ln 2$ si $A = \mathrm{e}^{\mathrm{e}}$ et $B = \mathrm{e}^{\mathrm{e}^2}$ (recall that : $\ln \mathrm{e} = 1$).
(Hint : $(\ln u)' = u'/u$ where u is a differentiable function that one has to choose carefully)

Nota bene : Réponses concises + trêve de bla-bla = temps épargné + correcteur bienveillant