

1. Consider the sequence  $u_0 = 1$ ,  $u_{n+1} = 3u_n + 2$ .
  - (1) Compute  $u_1, u_2, u_3, u_4, u_5$ .
  - (2) Show that  $u_n > 10$ , for any  $n > 2$ .
  - (3) Find the formula of  $u_n$ , as a function of  $n$ .
  - (4) Does the sequence  $x_n = 1/u_n$  converge?
  
2. Consider points  $A = (1, 1)$ ,  $B = (2, 3)$ ,  $C = (4, 5)$ .
  - (1) Are  $A, B, C$  on the same line?
  - (2) Find the distance from  $(0, 0)$  to the line  $AC$ .
  - (3) Compute the angles of the triangle  $ABC$ .
  - (4) Compute the area of the triangle  $ABC$ .
  
3. Let  $K = \sin(3x)$ .
  - (1) Express  $K$  as a function of  $\sin(x)$ ,  $\cos(x)$ .
  - (2) Express  $K$  as a function of  $\sin(x)$ .
  - (3) Express  $K$  as a function of  $\cos(x)$ . Discussion.
  - (4) Express  $K$  as a function of  $\tan(x/2)$ .
  
4. Let  $P$  be the plane  $x + y + 2z + 3 = 0$ .
  - (1) Does  $P$  pass through the point  $(2, -3, 1)$ ?
  - (2) Find the distance from  $(0, 0, 0)$  to the plane  $P$ .
  - (3) Find the intersection of  $P$  with the plane  $x + 2y + z = 0$ .
  - (4) Find the intersection of  $P$  with the sphere centered at  $(2, 2, 2)$ , of radius 1.
  
5. Consider the matrix  $A = \begin{pmatrix} 6 & -3 \\ 4 & -1 \end{pmatrix}$ .
  - (1) Find the characteristic polynomial of  $A$ .
  - (2) Find the eigenvalues and eigenvectors of  $A$ .
  - (3) Express  $v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  as a combination of eigenvalues of  $A$ .
  - (4) Compute explicitly (give the numeric answer) the vector  $A^5 v$ .

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$\implies$  justify all the answers