

Examen de Mathématiques pour les Sciences

Examen du 14 juin 2010 - 2^{ème} session

Durée : 1h30

Documents, mobiles and calculators are not allowed

The number of points is just indicative

Exercise 1. (4 points) Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{pmatrix} -4 & 3 \\ -2 & 1 \end{pmatrix}.$$

Exercise 2. (4 points)

(a) Give $DL_4(0)$ (Taylor expansion at order 4 in zero) of the following functions:

$$f(x) = \cos^2(x), \quad g(x) = \sin(2x) \cdot \cos(x).$$

(b) Compute the following limits:

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}, \quad \lim_{x \rightarrow 0} \frac{e^x \cos(2x) - (x + 1)}{\cos(x) - 1}.$$

Exercise 3. (4 points) Let

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Among the following matrix products compute those which are well defined:

$A \cdot {}^t A$, $A \cdot B$, ${}^t A \cdot B$, B^2 et $(A \cdot {}^t A \cdot B) \cdot (B \cdot A \cdot {}^t A)$, where ${}^t A$ denote the transposed matrix of A .

Exercise 4. (4 points) Find the orthogonal projection of the point $P = (4; 5) \in \mathbb{R}^2$ on the straight line ℓ given by its parametric representation :

$$x = 4t + 1, \quad y = 1 - 2t, \quad t \in \mathbb{R}. \quad (1)$$

Make a drawing and compute also the distance between P and ℓ .

Exercise 5. (4 points) Show that

$$\sum_{k=1}^n (k^2 + 1) = \frac{n^3}{3} + \frac{n^2}{2} + \frac{7n}{6}. \quad (2)$$

Indication: Use a proof by induction.